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Comments on "Contoured double cantilever beam specimens for fracture toughness measurement of adhesive joints"

Mai [1] introduces some confusion regarding the design and use of contoured double cantilever beam (CDCB) specimens which should be clarified. The major confusion with respect to this work centres about the use of the CDCB specimen in a range of crack lengths beyond the linear compliance region. Equation 1 is only applicable over the linear region. When the uncracked ligament, $(w-a)$ or b is too short, the back end of the specimen begins to control the compliance. This begins to occur when:

$$1.25\,h \leqslant b \leqslant 1.5\,h,$$

and whenever b is less than h, only the back end controls.

Unfortunately, the paper was quite difficult to understand because of nomenclature and definitions peculiar to this paper and different to those used in commerce (ASTM) for metallic systems as well as adhesives.

Mai's equation 3 states:

$$R = -\frac{U^2}{2t}\frac{\partial}{\partial a}\left(\frac{x}{U}\right) = -\frac{U^2}{2t}P; \quad \text{for} \quad a > 3\,\text{in.}$$

This equation comes from Gurney and Hunt [2] which is a 1971 reference. In an earlier (1960) paper by Irwin [3] this expression for unit thickness is given as:

$$\left(\frac{\partial U}{\partial c}\right) = \mathcal{G} = \frac{1}{2}\frac{F}{M}\left(\frac{\partial F}{\partial c}\right)_{e\ \text{fixed}},$$

where F is the force $= Me$, e is elongation, c the crack length, M the spring modulus, U the total energy, $(\partial F/\partial c)$ the rate of change of force with crack length, and $\mathcal{G}$ the strain energy release rate. Therefore, in terms of thickness, B, we can rewrite the above equation

$$\mathcal{G} = \frac{e^2}{2B}\frac{\partial M}{\partial c}.$$

Common nomenclature today would be:

$$\mathcal{G} = \frac{\Delta^2}{2B}\frac{\partial(1/C)}{\partial a},$$

where Δ is the displacement of the load, C the compliance $= 1/M = \Delta/P$, and a the crack length.

Converting Mai's paper to common nomenclature we see: $W = \Delta$, $t = B$, $X = P$, $P = [\partial(1/c)/\partial a] =$ constant. We believe it would have been less confusing to use standard ASTM E-24 nomenclature and to reference the work which provides the insight for the body of work known as fracture mechanics.

In addition to the two areas mentioned above, there are a number of confusing and either incomplete or wrong statements made in the text. Mai's Equation 1 reads:

$$R = \frac{4X^2}{Et^2}m \qquad (1)$$

where X is the fracture load, and t the adhesive thickness; and his Equation 2 reads:

$$m = \left(\frac{3a^2}{h^3} + \frac{1}{h}\right) \qquad (2)$$

where h is the height of the beam at a given crack length.

In an explanation of this equation, Mai points out that m is exact for slender CDCB adherends, but not correct for stiff ones ($m = 90$ versus $m = 4\,\text{in.}^{-1}$). He then states a modified Equation 1 based supposedly on crack tip effects and departure from beam theory:

$$R = \frac{4X^2}{Et^2}m'.$$

It should be pointed out that only the equation derived from compliance measurements (using m')

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is correct. This equation, having nothing to do with "crack tip effects", is always derived from experiment and the use of m is to design a beam shape for constant dC/da within a given range of a only. The author has also defined t incorrectly. This is not the adhesive thickness but the *bond width* denoted B by the ASTM D-14 [4].

$$\mathcal{G}_{IC} = \frac{4(L_{max})^2}{EB_n}\frac{8}{Eb}m',$$

where L_{max} is the load at the point of crack initiation, E the elastic modulus of the adherends, B the adherend width, B_n the bond line width, m' the replacement for m based on compliance calibrations.

Mai states that "The profiles of cantilever beams are designed such that for quasi-static crack propagation to occur the fracture load (X) is invariant with crack length (a) if R is a constant". This nomenclature is different from the ASTM RP D3433-75 [4] for both fracturing and for all Equation 1 terms except E and m'. It would help the reader if the standard nomenclature had been used.

Some additional points are: (1) p. 572, right-hand column, two lines up, $a \leqslant 3$ should be $a > 3$; and (2) the units used in Fig. 2 for fracture toughness are $kg\,cm^{-1}$; in the SI system of units they should be $kJ\,m^{-2}$.

Further confusion is added when one looks at the data collected. For example, the starter crack, a, is 3 in., at which point h is 2 in., b is 3 in. and $b = 1.5h$. This means that the crack is at the boundary of the linear compliance section. Never-the less, the record of Fig. 2 shows a small amount of constant-load crack extension. If one calculates the $\mathcal{G}_{IC}$ value using metric units we obtain:

$$\mathcal{G}_{IC} = \frac{4P^2}{EB^2}m' = \frac{4(18.35)^2\,(2.205)(2.146)}{0.5\;10^6\;(0.25)^2} = 0.26\,kg\,cm^{-1}.$$

This value is at odds with so called "irreversible work area" reported here (e.g. 0.51 to 0.61 $kg\,cm^{-1}$). The irreversible work area can be calculated using the methods shown by Irwin [3] as:

$$\mathcal{G} = \frac{1}{2}\,\frac{18.35\,kg\,(1/7)\,mm}{3.6\,mm \times 0.635\,mm} = 0.57\,kg\,cm^{-1}.$$

This is in agreement with Mai; however, there is a substantial discrepancy between this value and the one calculated from the initial crack length.

Before Mai can expect this method of $\mathcal{G}$ measurement to gain acceptance in the testing of adhesive bonds, he should point out the advantages and verify the values calculated, neither of which was done in this communication.

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Reply to "Comments on 'CDCB specimens for fracture toughness measurement of adhesive joints'"

I wish to thank Dr S. Mostovoy for his useful comments on the communication referred to above. However, certain points must be clarified here. For personal preference, I have chosen to present the paper within the framework of the now well-known "quasi-static crack propagation" approach which was pioneered by Professor C. Gurney; also, because of the range of readers and the nature of the *Journal*, I have not used standard ASTM-E24 nomenclature and definitions, since the conversion of one set of symbols to another should not be difficult. The errata (i.e. $a > 3$ in. and t, the thickness of the adhesive joint) as pointed out by Dr Mostovoy were unfortunately not detected during proof reading; the metric unit for fracture toughness (i.e. $kg\,cm^{-1}$) may be con-

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veniently converted to the S.I. unit (i.e. kJ m^{-2}) by multiplying by 0.98.

Dr Mostovoy quoted the following expression for the strain energy release rate (G_{Ic}):

$$G_{Ic} = \frac{4(L_{max})^2}{EB_n}\left(\frac{8}{Eb}\right)m' \quad (1)$$

where L_{max}, E, B_n, etc, have already been defined in his comments. However, it seems that, as given, the right-hand side expression of Equation 1 does not yield the correct dimension for G_{Ic},

The last point raised by Dr Mostovoy concerned the different G_{Ic} results as obtained from the equation, $G_{Ic} = (4P^2/EB^2)m'$, and that calculated by Gurney's irreversible work area method. When $a = 3$ in., and $h = 2$ in., $b = 2.5$ in.; so that $b = 1.25h$ and not $1.5h$ as assumed. Thus, the starter crack is just beyond the linear compliance section. Although the equation for fracture toughness, $G_{Ic} = (4P^2/EB^2)m'$, may still be used for this region of constant load crack extension, care must be taken that the appropriate value of m' be employed because its magnitude now depends on the crack length. It should perhaps be mentioned that in many other similar tests, this constant load crack extension behaviour is not observed. Without measuring E and m', estimates of G_{Ic} may be obtained directly from

$$G_{Ic} = \frac{P^2}{2}\frac{d}{dA}\left(\frac{u}{P}\right) \quad (2)$$

where A is the crack area, u displacement and P the applied load. Note that in this way, $(d/dA)(u/P) = 8m'/EB^2$. Since, as shown by experimental calibration, $P/u = (319.7 - 40.3A)$ for $4 < A < 7.6$, we have from Equation 2,

$$G_{Ic} = \frac{20.15P^2}{(319.7 - 40.3A)^2}. \quad (3)$$

Thus at $A = 4.84\ \text{cm}^2$, $P = 18.4$ kg, $G_{Ic} = 0.44\ \text{kg cm}^{-1}$; and at $A = 5.07\ \text{cm}^2$, $P = 18.4$ kg, $G_{Ic} = 0.51\ \text{kg cm}^{-1}$. These G_{Ic} values correspond to the beginning and end of the constant load crack extension and agree fairly well with that calculated from the irreversible work area method (i.e. $0.51\ \text{kg cm}^{-1}$). It should also be noted that the Young's modulus (E) of the adherend is 3 to 3.5 $\times 10^5\ \text{lbf in.}^{-2}$ and not $5 \times 10^5\ \text{lbf in.}^{-2}$. Thus, with due allowance for this, the value for G_{Ic} at the initial crack length is increased from 0.26 to $0.44\ \text{kg cm}^{-1}$, which agrees with that calculated from Equation 3.

It must be emphasized that the original intention of the communication is to bring out the fact that for CDCB specimens with $m = 4\ \text{in.}^{-1}$ and crack lengths exceeding 3 in., fracture takes place at constant displacement and not at constant load as would be expected for $a < 3$ in.. This observation not previously reported has been proven experimentally both for an adhesive joint (Fig. 2) and for PMMA (Fig. 3). The fracture toughness values calculated from $G_{Ic} = 20.15u^2$ for both materials agree closely with those determined from the irreversible work area method of Gurney. For example, consider Fig. 2 of original paper (and neglect the first crack area increment at constant load), the calculated G_{Ic} values (i.e. $G_{Ic} = 20.15u^2$) of 0.55, 0.58, 0.61 and $0.61\ \text{kg cm}^{-1}$ compare favourably with those estimated from Gurney's method of 0.554, 0.584, 0.61 and 0.523 kg cm^{-1}, respectively.

As mentioned in the original communication, such CDCB specimens with large crack lengths may be advantageously used in two situations, where previously load controlled operations become displacement controlled:

(1) In stress corrosion studies of adhesive joints where $(G, \dot{a})$ data are to be collected. Since G varies directly as u^2, a constant applied G can be maintained simply by wedge opening to a given displacement instead of hanging dead weights to the specimen. Crack velocities ($\dot{a}$) may then be easily measured as a function of the applied G.

(2) In fatigue testing of polymers and adhesive joints performed in a screw-driven testing machine where displacement control is easier monitored than load control. By cycling at a given Δu and hence a given ΔG, the corresponding crack growth per cycle (da/dN) may be measured. As an illustration, Fig. 1 shows the crack growth data of PMMA immersed in a light lubrication oil for several ΔG levels. These results were obtained from the CDCB specimens referred to in the communication and with starter cracks greater than 3.5 in.. Fig. 2 shows the relationship between $\log(da/dN)$ and $\log(\Delta G)$, which fits well the fatigue crack growth equation

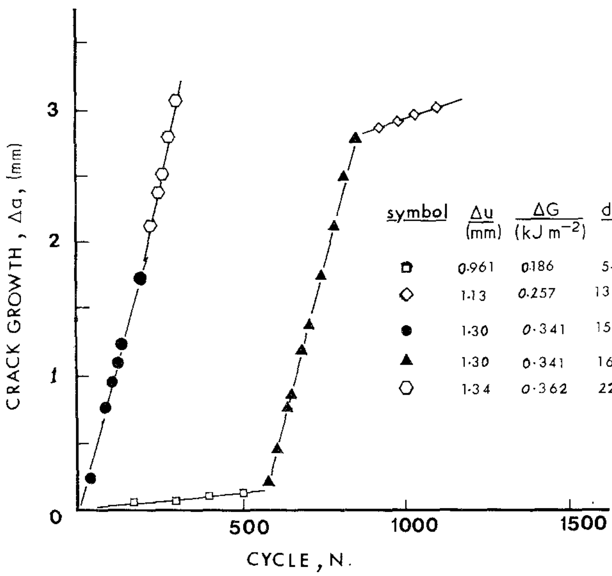

Figure 1 Crack growth of PMMA in a light lubrication oil at various ΔG levels.

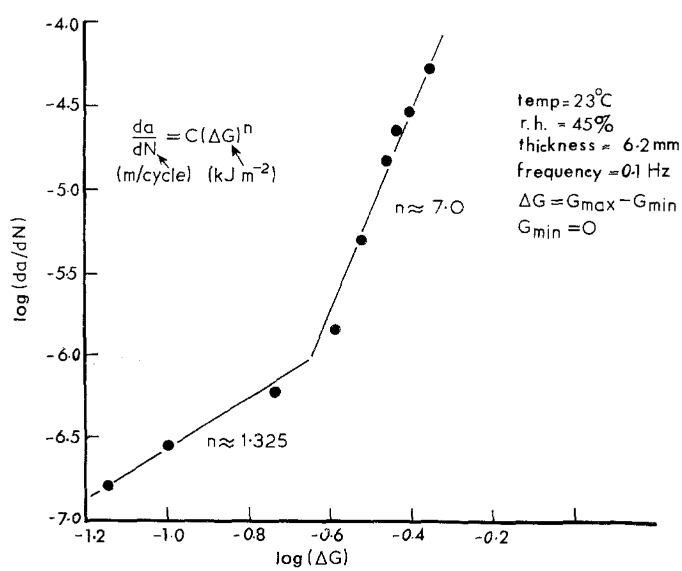

Figure 2 Relationship between log $(\mathrm{d}a/\mathrm{d}N)$ and log (ΔG) for environmental fatigue cracking of PMMA in a light lubrication oil.

$$\mathrm{d}a/\mathrm{d}N = C(\Delta G)^n \quad (4)$$

where n as shown is either 1.325 or 7.0 and C is a constant. These experimental data also agree closely with those obtained from SEN specimens.

Finally, I should add that the suggested method of fracture toughness measurement of adhesive joints is pertinent only for CDCB specimens with $m = 4\,\text{in.}^{-1}$ and large crack lengths (which are beyond the linear compliance section). It is not in any way meant to replace the already accepted test method developed by Dr Mostovoy and his co-workers.

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